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Enhanced power breathing soliton in communication systems with dispersion management

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A simple analytical description of the breathing pulse dynamics in optical transmission systems with dispersion management is presented. We demonstrate that a variational approach suggested in our previous works is an effective way to describe all features of the dispersion-managed soliton observed in numerical simulations and experiments. The developed method can be used for communication system design. [S1063-651X(97)50311-4]

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Increasing demand for telecommunications services stimulates research in the field of high-bit-rate optical data transmission. Research interest is directed mostly toward two main goals: development of long-haul optical amplifier systems with a transmission capacity of many tens of Gbit/s and the upgrade of existing fiber networks. Among other techniques the dispersion management is proving to be an efficient and promising method to be used both in ultralong transmission and for upgrading installed links at high bit rates. This Rapid Communication discusses a useful and effective analytical approach to describe optical pulse propagation in fiber lines with dispersion management. This problem recently became a key topic in optical high-bit-rate transmission research. The dispersion management is a well-established technique for ''linear'' signal transmission. We use here term linear transmission for the approaches in which the detrimental effects of dispersion and nonlinearity do not balance each other as they do in the soliton transmission. Three major factors limit linear transmission performance of an amplifier system: chromatic dispersion, nonlinearity, and noise. Optimization of the system performance in the case of the linear transmission requires minimization of the chromatic dispersion of the line. The dispersion compensation technique has been used successfully both in long-haul communication systems and in the existing terrestrial optical links, most of which are based on standard telecommunication fiber with large dispersion in the second optical window (at 1.55 μm). The basic optical-pulse equalizing system consists of a transmission fiber [standard monomode fiber (SMF) or dispersion-shifted fiber] and equalizer fiber with the opposite dispersion [e.g., dispersion compensating fiber (DCF)] [1]. In the linear regime, compensation of dispersion aims to prevent dispersive broadening of the pulse. Dispersion broadening of a signal in the transmission fiber is compensated by a pulse compression in the compensating fiber. An additional advantage is that the impact of the four-wave mixing on a signal transmission is suppressed due to the reduction of the efficiency of the phase matching. It has been found recently that the dispersion management is also a very promising way to increase the transmission capacity of a soliton-based communication line [2–12]. Energy of the dispersion-managed soliton is enhanced in comparison with a fundamental soliton [soliton solution of the nonlinear Schrödinger equation (NLSE)] corresponding to the same residual dispersion. This energy enhancement allows us to increase the signal-to-noise ratio with substantial improvement of system performance. The breathing soliton propagating in the link with dispersion compensation is chirped in contrast to the NLSE soliton. As a matter of fact, the breathing soliton presents a new type of a nonlinear carrier of information in optical fiber links. Being a stable solitary wave that realizes a delicate balance between varying dispersion and nonlinearity the dispersion-managed soliton both is of interest for the fundamental nonlinear science and is of great practical importance.

In this Rapid Communication we demonstrate that a variational approach applied in [4,7] to the dispersion management (see also recent works [8,10,13] and a nice paper [9]) is an extremely effective tool that allows us to explain all features of the dispersion-managed soliton observed in numerical simulations, [2–5] This approach provides a clear physical picture of the pulse evolution in the transmission line under the combined action of the nonlinearity, varying dispersion, fiber loss, and periodic amplification. Developed method can be used for optical communication system design.

Transmission of optical signal in fiber link with dispersion compensation is governed by the following basic model:

$$i\Psi_z + d(z)\Psi_{tt} + \frac{L}{Z_{\rm NL}}|\Psi|^2\Psi = iG(z)\Psi;$$

$$G(z)\Psi = L\Big(-\gamma + r\sum_{k=1}^{N}\delta(z-z_k)\Big)\Psi. \qquad (1)$$

We use here the following normalization: z is normalized to a dispersion map length L (in km); time is measured in t_0 (in picoseconds) that should be specified for each concrete problem; an envelope of the electric field $E=E(T,Z)$ is normalized to the power P_0: $|E|^2 = P_0|\Psi|^2$; $Z_{\rm NL}=1/(\sigma P_0)$—nonlinear length; σ is the coefficient of the nonlinearity; γ describes fiber losses, r is the amplification coefficient, z_k are the amplifiers locations; normalized dispersion

$d(z)=-L\beta_2/(2t_0^2)$; here β_2 is the group velocity dispersion varying periodically with z. The amplification period can be different from the dispersion compensation period. Though the main results of this work will be formulated in a general form and can be used for arbitrary dispersion map; in the illustrations without loss of generality we use symmetrical dispersion map for lossless system studied in [5]. As was pointed out in [5] the inclusion of periodic amplification and dispersion compensation can be handled as separate problems, provided that amplification distance is substantially different from the period of dispersion map. Equation (1) can be written in the Lagrangian form

$$S=\int L dt dz$$
$$=\int dt dz\left[\frac{i}{2}(\Psi\Psi_z^*-\Psi^*\Psi_z)+d(z)|\Psi_t|^2-\frac{c(z)}{2}|\Psi|^4\right], \tag{2}$$

where $c(z)=c_0\exp[2\int_0^z G(z')dz']$ with $c_0=P_0L\sigma$. In this short publication we focus on the discussion of the results rather than on the derivation of the key equations. Using a standard technique (see for details [14,4,7] and references therein) it can be shown that the asymptotic breathing dynamics of the central part of an optical pulse propagating in the system with dispersion management is described in the leading order by the following trial function:

$$\Psi(z,t)=\frac{Q(x)}{\sqrt{T(z)}}\exp\left(i\frac{M(z)}{T(z)}t^2+i\lambda(z)\right), \tag{3}$$

here $x=t/T(z)$ and evolution of $T(z)$ and $M(z)$ is given by

$$\frac{dT}{dz}=4d(z)M, \tag{4}$$

$$\frac{dM}{dz}=\frac{d(z)C_1}{T^3}-\frac{c(z)C_2}{T^2};\quad c(z)=c_0\exp\left(2\int_0^z G(z')dz'\right). \tag{5}$$

The constants C_1 and C_2 are related to a structural function $Q(x)$ through

$$C_1=\frac{\int|Q_x(x)|^2dx}{\int x^2|Q(x)|^2dx},\quad C_2=\frac{\int|Q(x)|^4dx}{\left(4\int x^2|Q(x)|^2dx\right)}. \tag{6}$$

Equations (4),(5) should be solved for specific dispersion and power maps with initial conditions corresponding to the input pulse. To keep at the beginning of each section pulse width normalized as above, we use the following initial conditions: $T(0)=1$ and $M(0)=M_0$. A pulse energy in the real world variables is given by

$$E=P_0t_0\int|Q(x)|^2dx. \tag{7}$$

For a specific choice of a structural function Q one can find the dependence of the soliton energy on the parameters of the system and input pulse. For instance, for input Gaussian pulse $Q(x)=A\exp(-0.5x^2)$ it is easy to find that $C_1=1$ and $C_2=A^2/(2\sqrt{2})$. Energy of the asymptotic pulse E is proportional to the C_2 and is given in this case by

$$E=P_0t_02\sqrt{2\pi}C_2. \tag{8}$$

Pulse width T_{FWHM} (full width at half maximum) is found as $T_{\mathrm{FWHM}}=1.665t_0$. Below we discuss how the shape of a dispersion-managed soliton can be described using the variational approach.

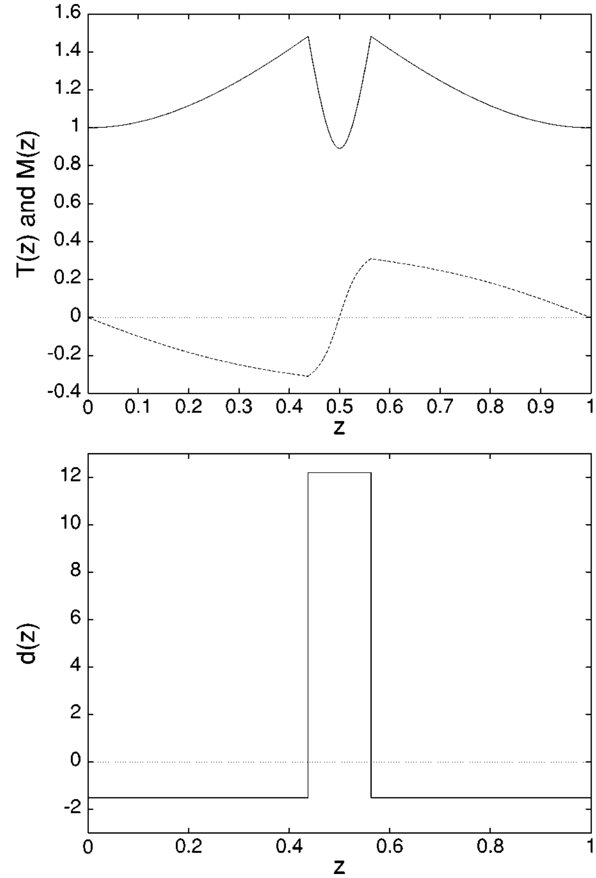

FIG. 1. Symmetrical dispersion map $d(z)$ and typical periodic solutions $T(z)$ and $M(z)$ for this map [initial conditions are $T(0)=1$, $M(0)=0$)]. Here map strength $K=3$, average dispersion $\langle d\rangle=0.2$, $C_1=0.5$.

Let us consider as an example a symmetrical dispersion map studied in [5]. A piece of a fiber with the dispersion $\beta_2^{(1)}>0$ [dispersion shifted fiber (DSF)] and length Z_c is followed by the compensating fiber with the dispersion $\beta_2^{(2)}<0$ (here SMF) and length $L-2Z_c$ in the center and at the end of the section it is placed symmetrically the same fiber as at the beginning. Considering SMF as the central fiber, we assume dispersion of the second piece to be $D_2\approx17$ ps/(nm$\times$km) at the operating wavelength 1.55μm. Strength of the map can be changed by varying dispersion of the DSF pieces. Periodic solutions corresponding to this map are plotted in Fig. 1. Recall that the normalized dispersion (indices are for different pieces of fiber) and the average dispersion are

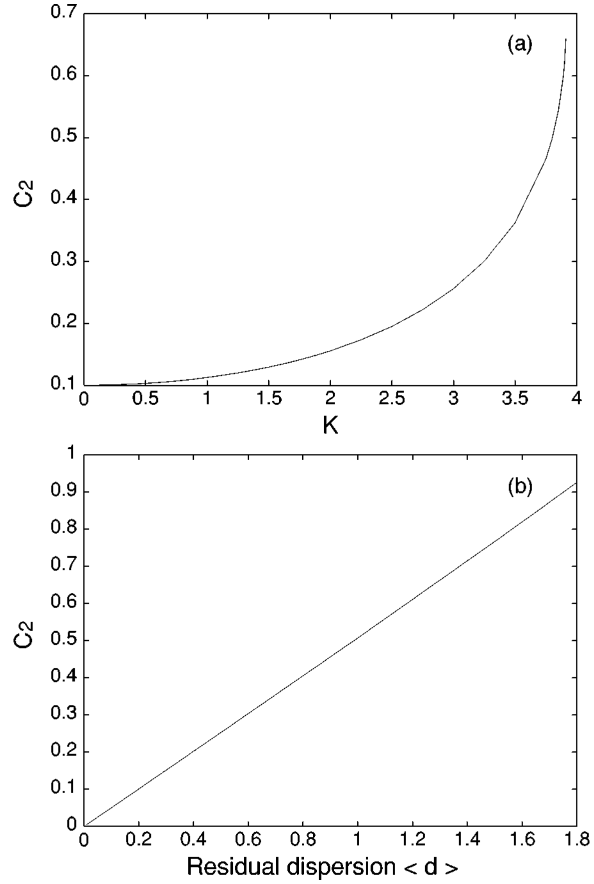

FIG. 2. Dependence of the coefficient C_2 that is proportional to the soliton energy on the effective strength of the map K (a) and average dispersion $\langle d \rangle$ (b). One can see enhancement of the energy with the increasing of the strength of the map. Here parameters of the map are: $C_1=0.5$, $\langle d \rangle=0.2$ in (a) and $K=0.2$ in (b).

$$d_k=-\frac{\beta_2^{(k)}}{2t_0^2}L;\quad \langle d \rangle=-\frac{\langle \beta_2 \rangle L}{2t_0^2}. \tag{9}$$

Figure 2 displays the dependence of the parameter C_2 (and consequently—the energy of an asymptotic pulse) on the parameter characterizing a variation of the dispersion (strength of the map) $K=[2Z_c\beta_2^{(1)}-(L-2Z_c)\beta_1^{(2)}]/(2t_0^2)$ and the residual dispersion $\langle d \rangle$. As can be seen from Fig. 2(a) the energy of the asymptotic pulse (that is proportional to C_2) increases with growth of the map strength K. Figure 2(b) shows that in this range of parameters asymptotic pulse energy linearly depends on the average dispersion. These results are in accordance with the empirical formula presented in [5] and analytical approach developed in [6]. A new interesting issue is that the strength of the considered map is bounded from above by some critical (cutoff) value of K. This is a feature of the specific structure of the map. For instance, if we consider as a central fiber a piece of DSF with $d_2=-d_1$ it is possible to find that there is no restriction in K. Mathematically, the occurrence of the critical K corresponds to the following. Using the considered map, we effectively cross a region of existence of the periodic solutions of Eqs. (4) and (5) in the plane (d_2,K) along the line $d_2=\text{const}$. This line intersects a right border line of the region of existence of the periodic solutions at some K_{cr}. Cut-

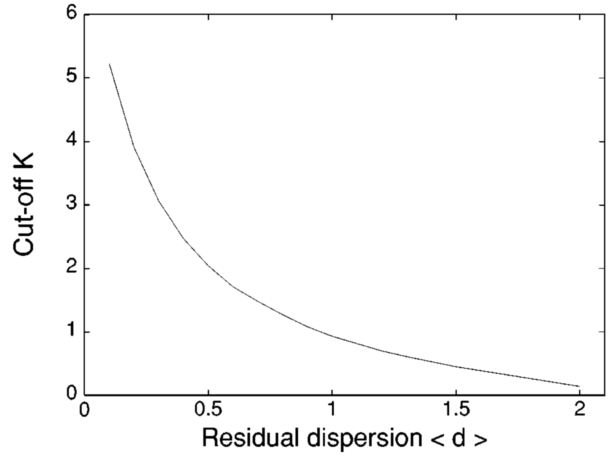

FIG. 3. Cutoff value of the parameter K versus residual dispersion; $C_1=0.5$.

off parameter K decreases with the increasing of the residual dispersion as shown in Fig. 3.

As it was first mentioned in [7] the variational approach can be used also to determine the shape of the dispersion-managed soliton. Consider a trial function (3) with periodic functions T and M satisfying Eqs. (4),(5) for some constant C_1 and C_2. Averaging over one period in z we get again a problem of the minimization of the functional S. The breathing soliton shape is found as a solution of the following equation:

$$-kQ+Q_{xx}+\frac{r_2}{r_1}Q^3-ax^2Q=0;\quad a=\left(C_1-C_2\frac{r_2}{r_1}\right). \tag{10}$$

Here k is a parameter of the soliton and r_1,r_2,C_1,C_2 are constants related to a dispersion map. The procedure to describe a structure of the asymptotic breathing soliton corresponding to a specific map is the following. First, we find periodic solutions of Eqs. (4),(5) T and M for arbitrary C_1. Then, it should be calculated coefficients $r_1=\langle d/T^2 \rangle$ and $r_2=\langle c/T \rangle$. The next step is to solve Eq. (10) for arbitrary k and C_1. For the dispersion compensating systems the parameter a depends on the characteristics of the dispersion map, pulse characteristics, and residual dispersion. In the limit $a=0$ solution is a soliton of the NLSE. Our analysis of different dispersion maps show that the parameter a in the simplest versions of the compensating systems similar to the ones studied in [2,5,7] is always negative. Negative a corresponds to the tunneling of the radiation from the central part of the dispersion-managed soliton. An effective potential in Eq. (10) is of the nontrapping type in this case. The typical solution of Eq. (10) is shown in Fig. 4. Localized solutions of Eq. (10) decay very slowly ($|Q|^2\rightarrow 1/x$ as $|x|\rightarrow\infty$). This steady-state solution can approximate the central part of the asymptotic solution in the nonstationary problem for small a. In this case a slow tunneling of the radiation from the main peak takes place due to nontrapping parabolic potential. To calculate energy of the main pulse and to define pulse width (for $a<0$) it is necessary to introduce some cutoff in time around the central peak. It should be noted that Eq. (10) has been derived in [11] by exact averaging of the master equation (1). Also this equation (with $a>0$) describes the

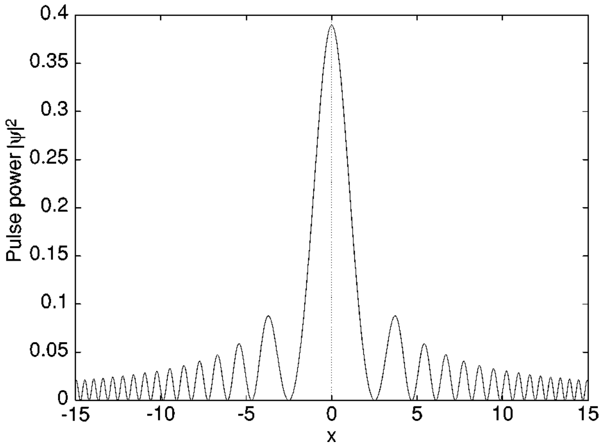

FIG. 4. Typical solution of Eq. (10) for the symmetrical dispersion map shown in Fig. 1. For the parameters of the map shown in Fig. 1 coefficient $a = -0.1815$ and $r_2/r_1 = 2.659$.

shape of the quasisoliton in the system with programmed chirp and dispersion considered in [12]. It has been found in [13] that using the additional grating at the end of the compensation cell allows us to operate in a regime with $a > 0$ and as a result to form a carrier pulse with Gaussian tails. It should be pointed out that although the variational approach describes with high accuracy the evolution of the central part of the pulse this description is limited due to shedding a part of the pulse energy into a dispersive wave during propagation. This shedding of the energy into a dispersive pedestal can be minimized by a proper choice of the input pulse parameters using the above results. Note that in the considered problem using the variational method is even more justified than, for instance, in the case of the NLSE. This is because an asymptotic breathing soliton has just the parabolic phase (in time) at the central part like in the trial function (3) and this is very different from the phase of a fundamental soliton The variational approach can be successfully used both for the description of the asymptotic state and the initial stage of input pulse evolution in the transmission systems with dispersion management [7–9,10,13].

In conclusion, we demonstrate that a variational approach is a very effective tool to describe breathing pulse dynamics in optical communication systems with dispersion management. This method presents a useful and effective analytical approach to describe optical pulse propagation in fiber lines with dispersion management. Soliton transmission system design and optimization can be effectively managed using this approach. Developed variational method allows us to calculate power enhancement for a dispersion-managed soliton. It is shown also that a shape of dispersion-managed pulse is given by the NLS equation with additional parabolic potential of nontrapping type.

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